# Forms of Approximate Radiation Transport

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# The Diffusion Approximation

- Taylor Expand Intensity in Angle

$$I = \frac{c}{4\pi} E + \frac{3}{4\pi} \mathbf{\Omega} \cdot \mathbf{F} \qquad \mathbf{F} = -\frac{c}{3\sigma_t} \nabla E$$

- Fast, robust, and accurate numerical solutions
- Flux can be larger than energy density
  - More stuff moving than is there to move
- Flux Limited Diffusion improves robustness
  - Limits flux so that it’s not larger than energy density
  - Many different flux limiters

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# Spherical Harmonics (PN)

- **Intensity expanded further in angle than diffusion**

$$I(\mathbf{r},\mathbf{\Omega},\varepsilon,t)=\frac{1}{\sqrt{4\pi}}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}E_l^m(\mathbf{r},\varepsilon,t)Y_l^m(\mathbf{\Omega})$$

- **Equivalent to discrete ordinates in one dimension**
- **Not as widely used**
  - **Some pitfalls not so well known**
  - **Some good algorithms, probably not the best**

# Discrete Ordinates (SN)

- Intensity assumed sum of delta functions
- Particles move only along certain directions
  - Not nearly as many as the infinite directions of the transport equation
- Well studied
  - Pitfalls known, some attempts to fix them
  - Some very fast, efficient algorithms

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# Monte Carlo

- NOT an approximation of the transport equation
- Builds up an average solution by simulating many individual particles
- Can model all the physics you want easily
- CPU time and memory intensive
- Can calculate estimate of error at any time
  - Run more particles if you need better answer for linear problems, but it will cost you

$$\mathrm{Error} = \alpha \frac{1}{\sqrt{N}}$$

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# A Line Source

- **Pulsed isotropic line source at *t=0* in vacuum**
- **Analytic transport solution**

$$E_{\text{transport}} = \frac{E_0}{2\pi} \frac{\mathrm{h}(ct - r)}{ct\sqrt{c^2t^2 - r^2}}$$

- **Analytic PN solution has negative components**

$$E_{P_N} = \frac{E_0}{\pi} \sum_{\lambda_i \geq 0} r_i l_i \left[ \frac{\delta(r - \lambda_i t)}{\sqrt{\lambda_i^2 t^2 - r^2}} - \frac{\lambda_i t\, \mathrm{h}(\lambda_i t - r)}{(\lambda_i^2 t^2 - r^2)^{3/2}} \right]$$

- **Analytic SN solution is a function of *x* and *y***

$$E_{S_N} = E_0 \sum_i w_i \delta(\|\mathbf{x} - ct\mathbf{\Omega}_i\|)$$

- **Tests fundamental properties of approximations**

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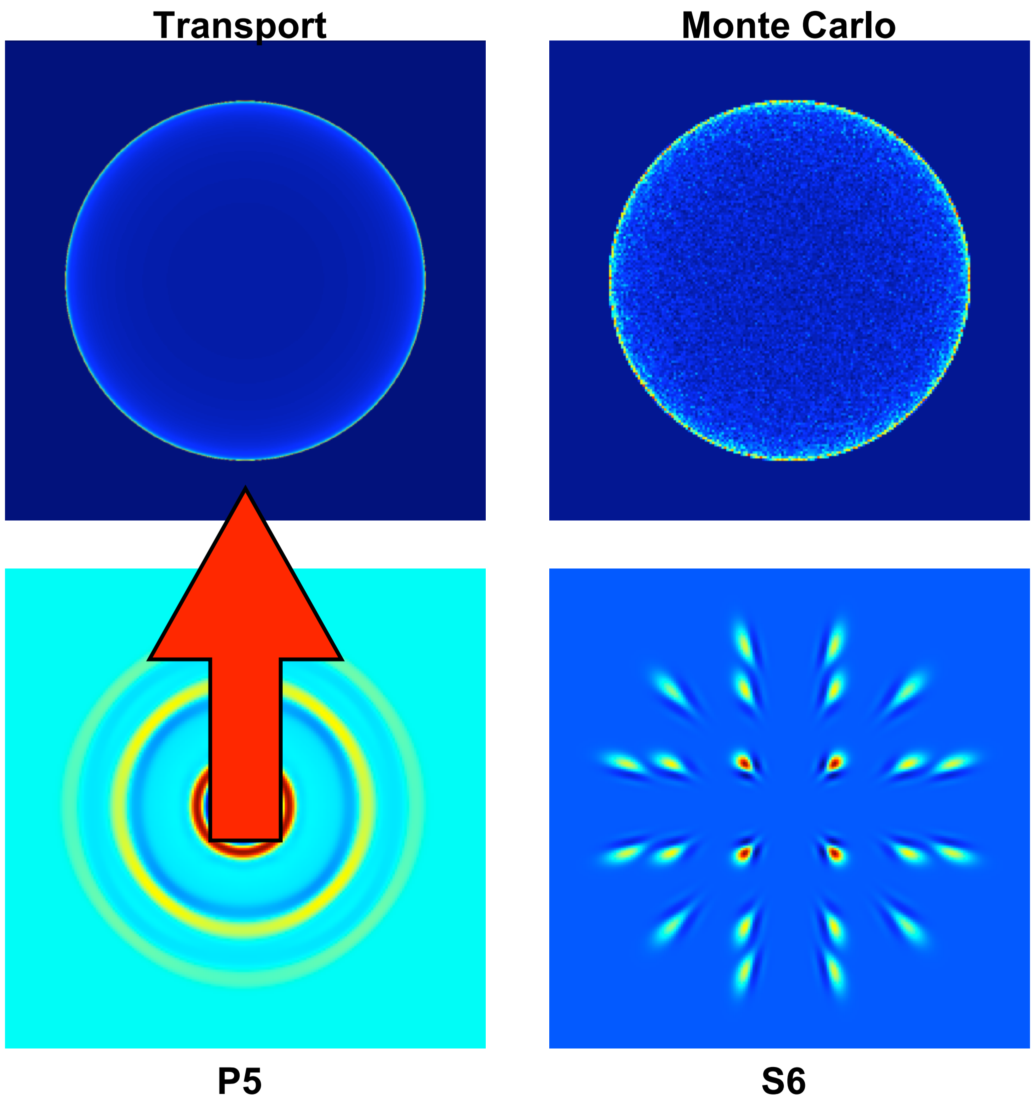
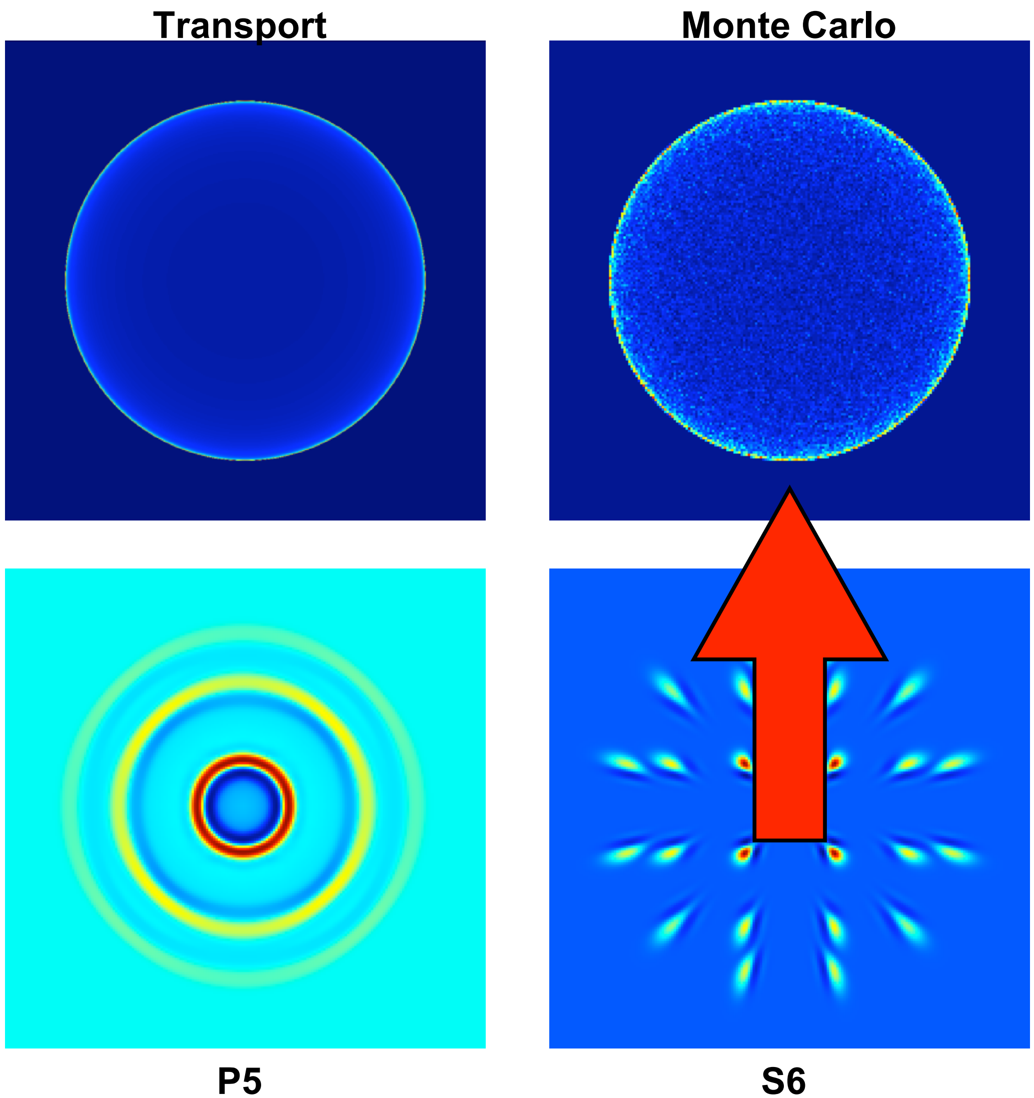
# Transport Solution




- Smooth interior
- Sharp discontinuity
- Particles limited by speed of light

# Monte Carlo




- Noisy
- Discontinuity spread out
- Particles limited by speed of light

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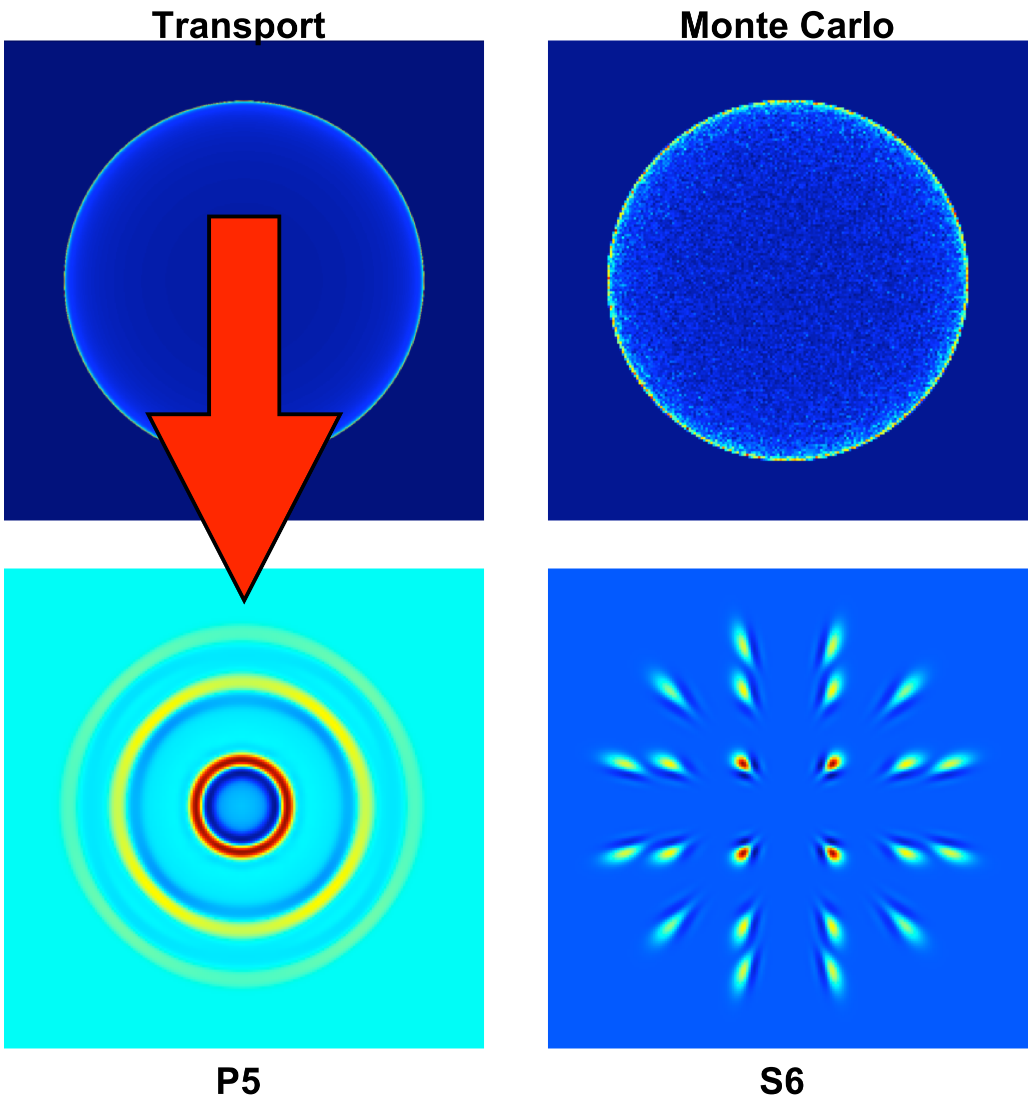
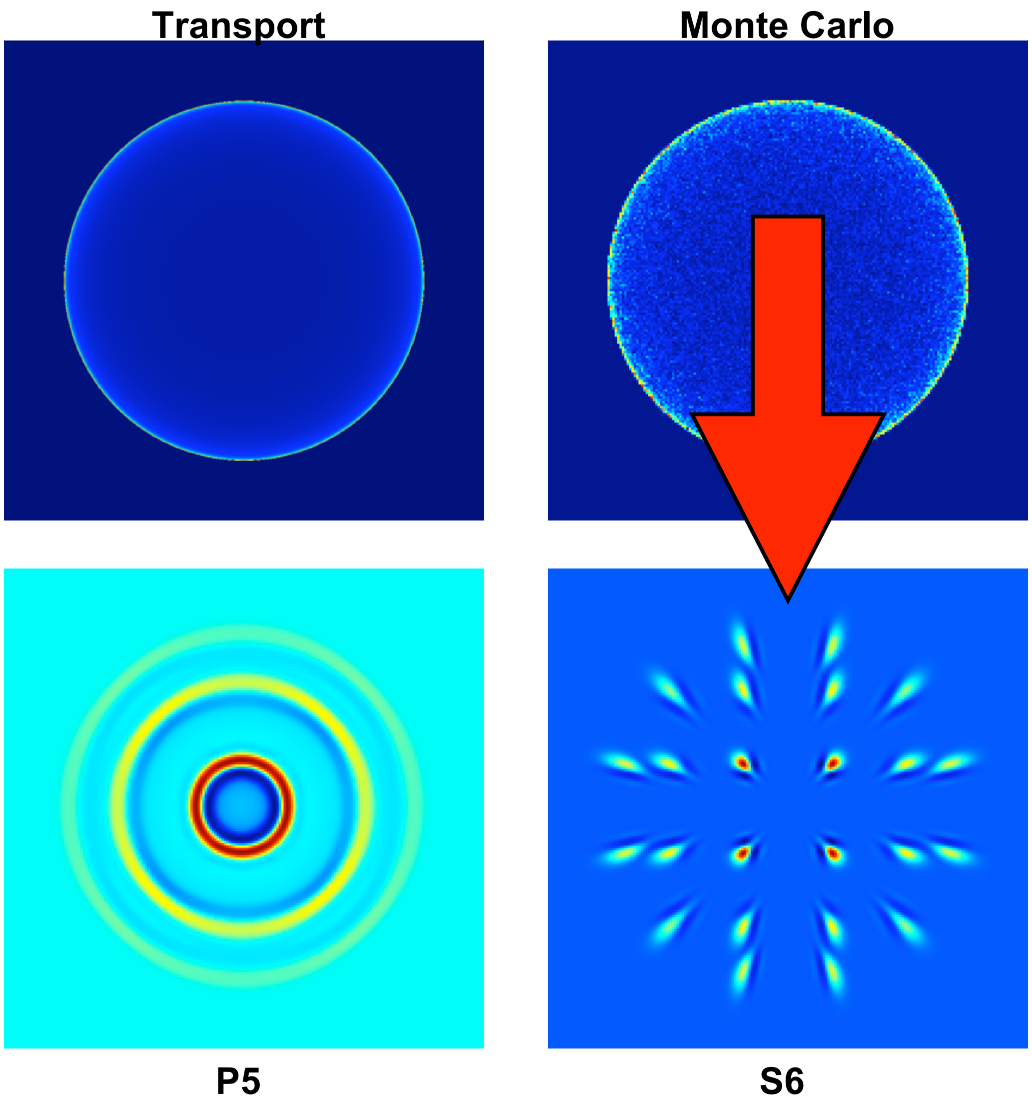
# Spherical Harmonics (P5)




- Particles move in three waves
- Negative regions behind waves
- Rotationally invariant
- Particles limited by speed of light
- VERY WRONG—not modeling reality well for this problem

# Discrete Ordinates (S6)




- Particles move along rays
- Nonnegative
- Particles limited by speed of light
- VERY WRONG—not modeling reality well for this problem

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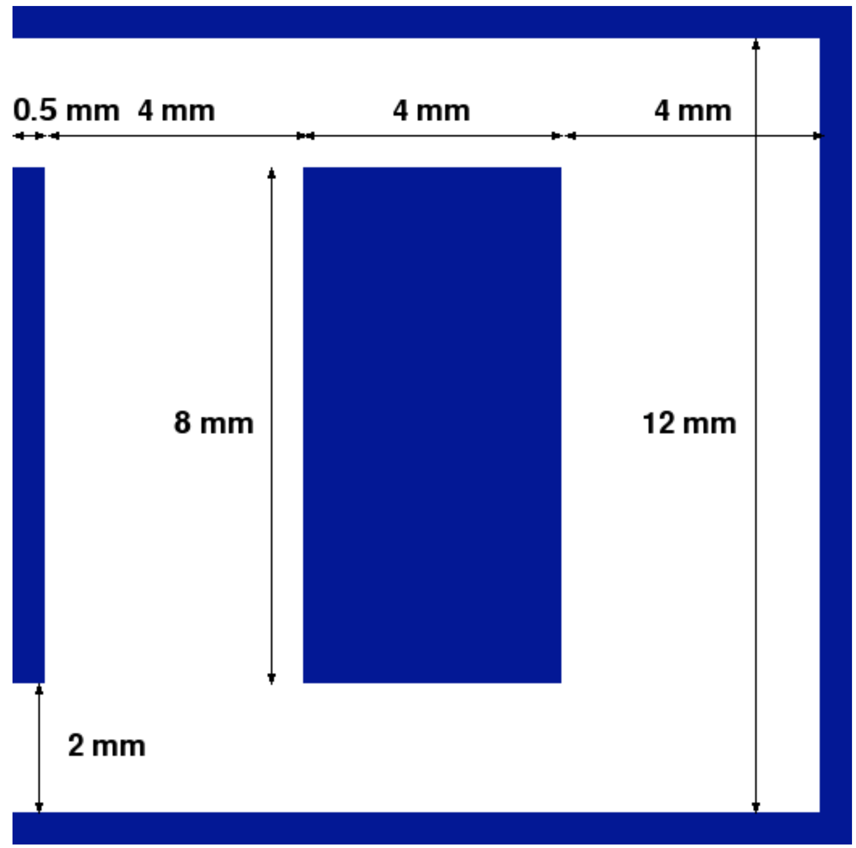
# A Hohlraum Problem

A test of radiation transport in a system with vacuum, material heating, and reemission.

- Cartesian hohlraum loosely bases on ICF capsule
- Absorption=100/cm in blue
- Vacuum in white
- Initial temperature 300K
- Source boundary condition on left, temperature 3.5e6K
- Color map proportional to radiation temperature

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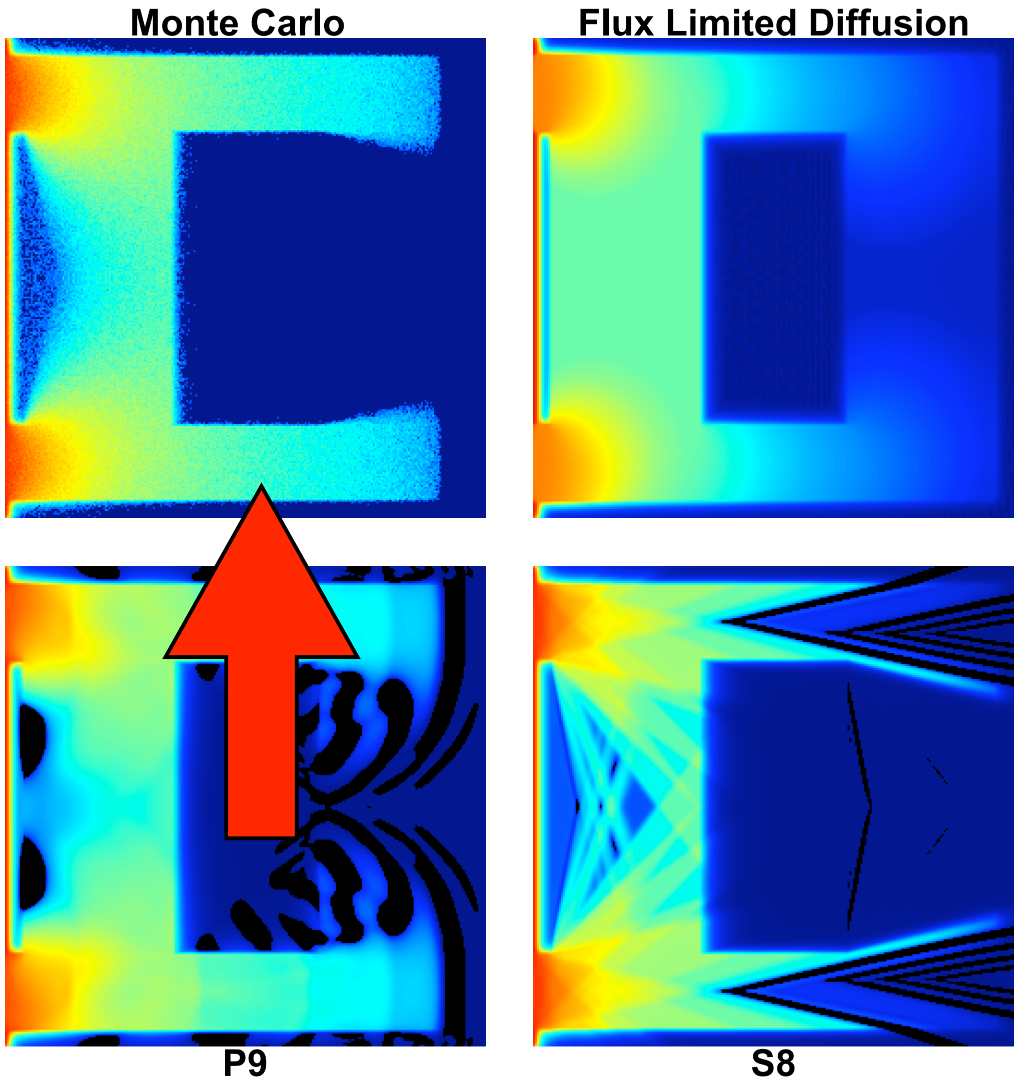
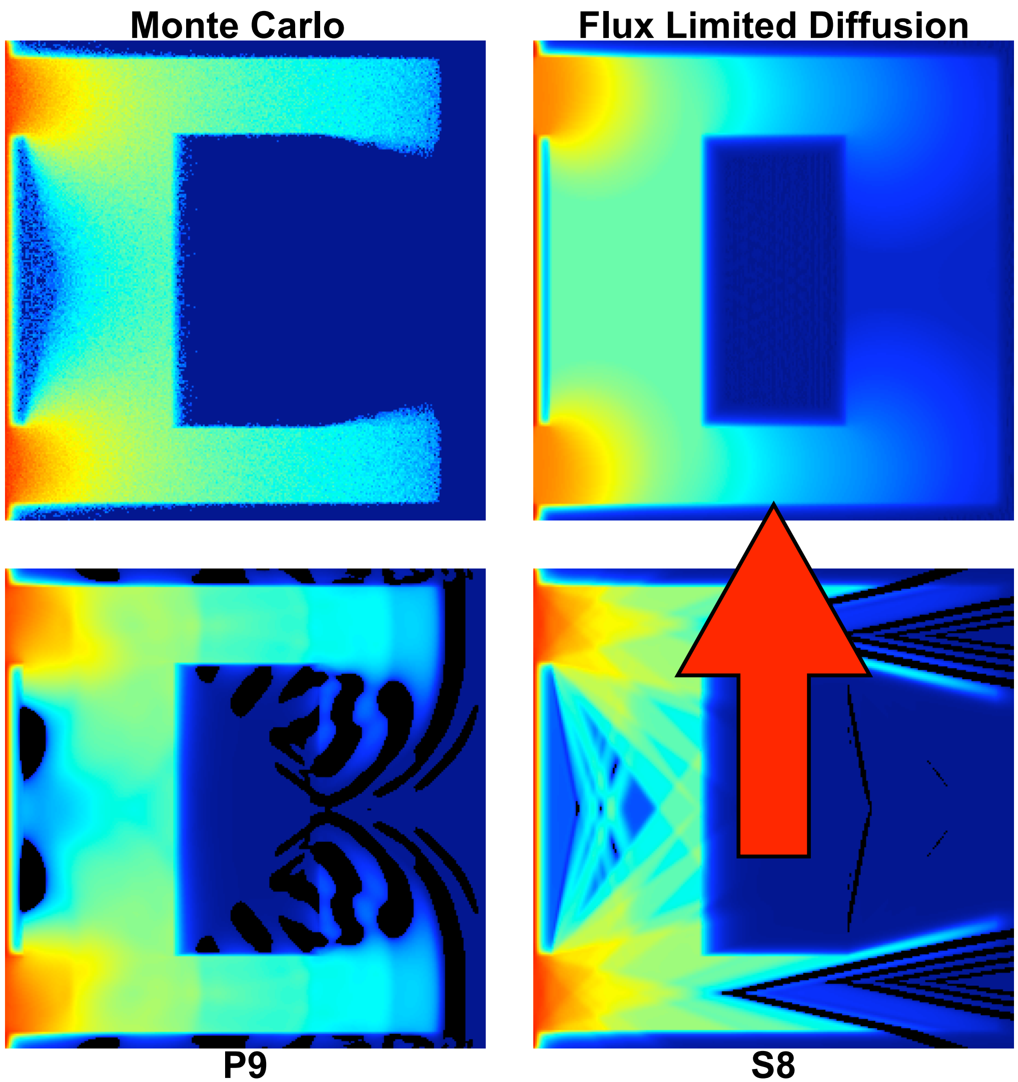
# Monte Carlo



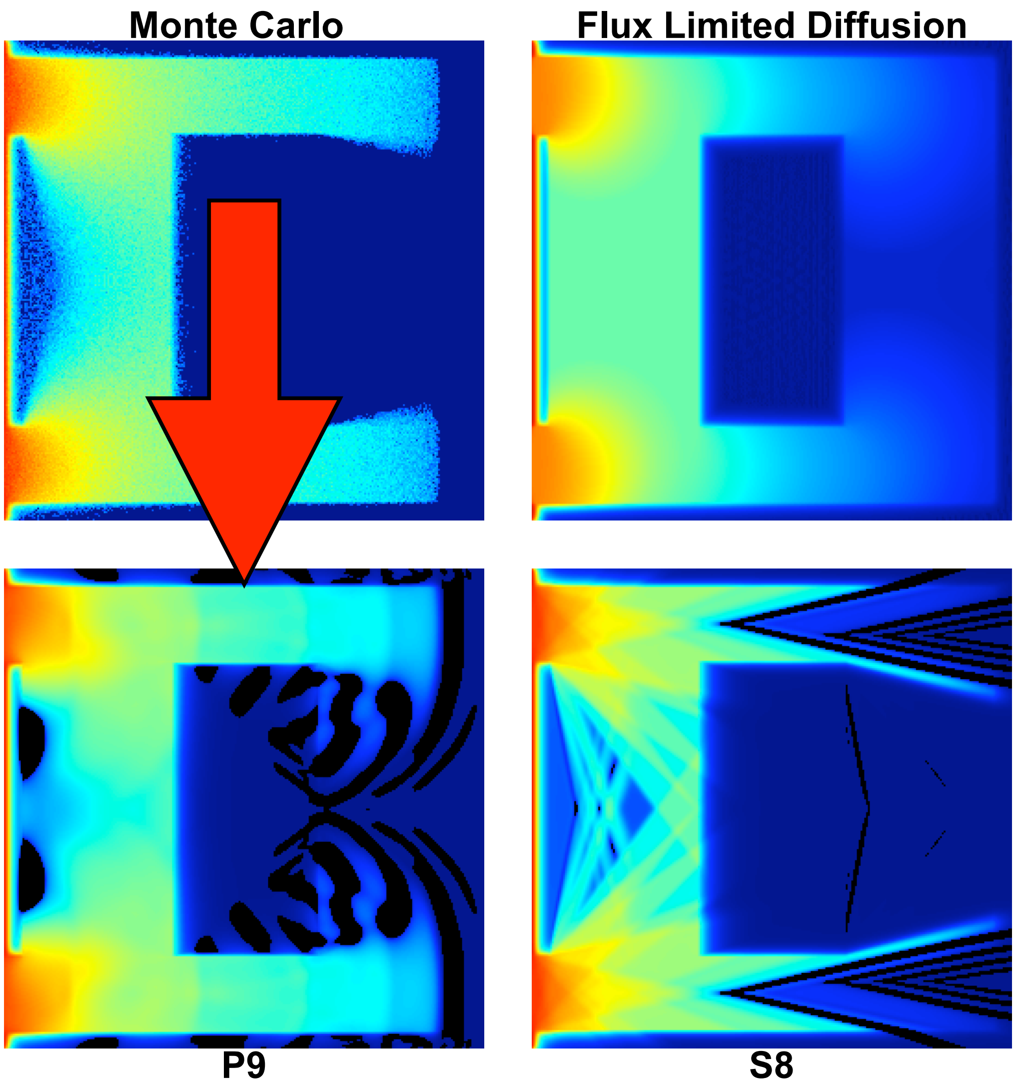

- Photons just reaching back wall
- Shadow behind shield on left
- Some Noise in radiation temperature (shown)
- Material temperature (not shown) is much smoother

# Flux Limited Diffusion




- Photons everywhere, even where they shouldn't be
- No shadow behind shield
- Too uniform

# Spherical Harmonics (P9)




- Wave effects to the left of shield and capsule
- Too many particles “bent” around corners
- Negative regions (black) are “compensating”

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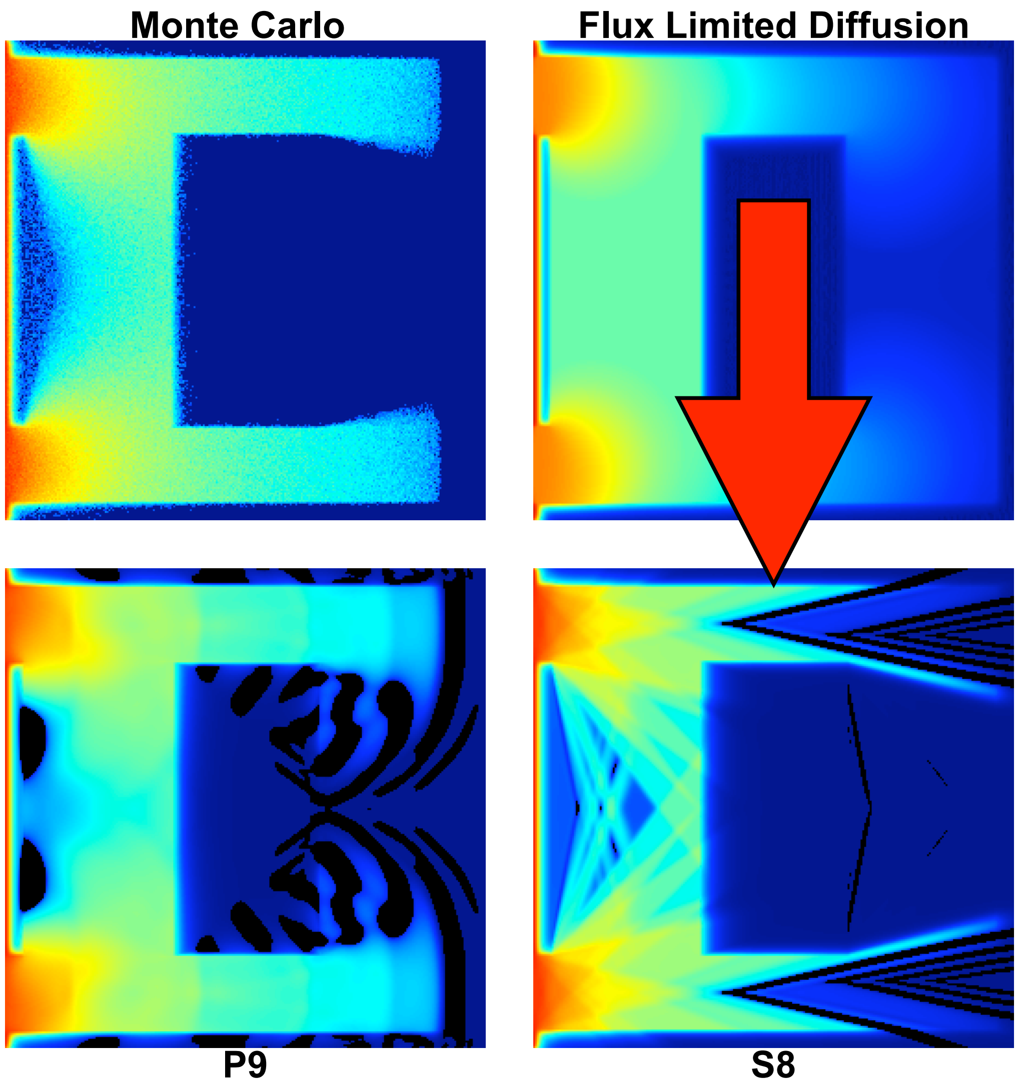
# Discrete Ordinates (S8)




- Ray effects dominate solution and persist to long times. Wave effects in PN tend to go away.
- Beam-like solutions causing material hot-spots.

# Conclusions

- Monte Carlo gives best looking results.  Theory and numerical methods well understood.
- Flux limited diffusion is a close second, especially when factoring in run time.  Theory and numerical methods are well understood.
- Spherical harmonics (PN) suffers from wave effects, but theses tend to go away at long times. Not very common, so not well understood.
- Discrete  ordinates (SN) suffers from ray effects which can persist even in steady state.  Lots of effort in this approximation, so its pitfalls are well known.